Precalculus

7-05 Rotated Conics

- Nonrotated conics form $Ax^2 + Cy^2 + Dx + Ey + F = 0$.
  - _____horizontal or vertical.
- Rotated conics form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.
  - _____horizontal or vertical
  - Bxy term prevents _____the _____to write the conics in standard form.

Classify Rotated Conics

- If the conic is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, then
  - If $B^2 - 4AC < 0 \rightarrow$ _____
  - If $B^2 - 4AC = 0 \rightarrow$ _____
  - If $B^2 - 4AC > 0 \rightarrow$ _____

Write Rotated Conics in Standard Form

Given a conic written as $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

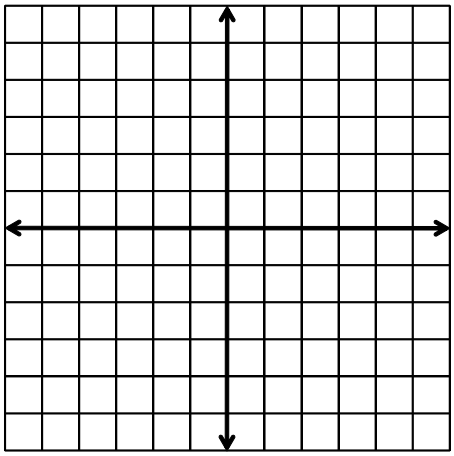
1. Find the angle of rotation using
$$\mathbf{\cot 2\theta = \frac{A - C}{B}}$$
where $0 < \theta < \frac{\pi}{2}$
2. Find $\sin\theta$ and $\cos\theta$.
   - If θ is a special angle, evaluate $\sin\theta$ and $\cos\theta$ directly.
   - If θ is not a special angle,
   a. Find $\cot 2\theta$.
   b. Reciprocal to find $\mathbf{\tan 2\theta}$.
   c. Use $\mathbf{1 + \tan^2 u = \sec^2 u}$ to find $\sec 2\theta$. (If $\tan 2\theta < 0$, then $\sec 2\theta < 0$.)
   d. Reciprocal to find $\mathbf{\cos 2\theta}$.
   e. Use the half-angle formulas to find $\sin\theta$ and $\cos\theta$.
   $$\mathbf{\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}} \text{ and } \mathbf{\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}}$$
3. Find the substitutions for x and y using
$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$
4. Make the substitutions and arrange the terms into standard form.

Graph a Rotated Conic

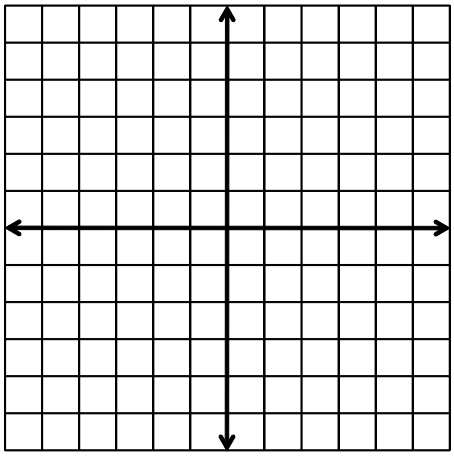
1. Draw the rotated _____.
2. Using the rotated axes, sketch the _____.

Write $xy = \frac{1}{2}$ in standard form

Sketch the graph of $x^2 + \sqrt{3}xy + 2y^2 - 2 = 0$.



Sketch the graph of $3x^2 + 2\sqrt{3}xy + y^2 + 2x - 2\sqrt{3}y = 0$.



Classify the graph, use the quadratic formula to solve for y, and use a graphing utility to graph the equation.

$3x^2 - 6xy + 3y^2 + 2y = 0$